Measuring the probability of a financial crisis

Robert F. Engle[a,1] and Tianyue Ruan[b,1]

[a]Stern School of Business, New York University, New York, NY 10012; and [b]NUS Business School, National University of Singapore, Singapore 119245

Contributed by Robert F. Engle, July 19, 2019 (sent for review March 11, 2019; reviewed by Tobias Adrian and J. Darrell Duffie)

When financial firms are undercapitalized, they are vulnerable to external shocks. The natural response to such vulnerability is to reduce leverage, and this can endogenously start a financial crisis. Excessive credit growth, the main cause of financial crises, is reflected in the undercapitalization of the financial sector. Market-based measures of systemic risk such as SRISK, which stands for systemic risk, enable monitoring how such weakness emerges and progresses in real time. In this paper, we develop quantitative estimates of the level of systemic risk in the financial sector that precipitates a financial crisis. Common approaches to reduce leverage correspond to specific scaling of systemic risk measures. In an econometric framework that recognizes financial crises represent left tail events for the economy, we estimate the relationship between SRISK and the financial crisis severity for 23 developed countries. We develop a probability of crisis measure and an SRISK capacity measure based on our estimates. Our analysis highlights the important global externality whereby the risk of a crisis in one country is strongly influenced by the undercapitalization of the rest of the world.

financial stability | systemic risk | probability of crisis

When financial institutions are undercapitalized, they are vulnerable to external shocks. Even more importantly, they may become the source of disruptive shocks to the broader economy through their deleveraging behaviors to regain adequate capitalization. When undercapitalization is extreme, these endogenous shocks become sufficient to cause an economic downturn. The process by which undercapitalization leads to a financial crisis has been widely studied in the theoretical macro-finance literature and to some extent in the empirical literature. These models feature asset fire sales and credit rationing in various forms. We build on existing empirical measures and come up with quantitative estimates of how much systemic risk it takes to generate a financial crisis.

Our approach is motivated by the observation that excessive credit growth, a main cause of financial crises, is reflected in the undercapitalization of the financial sector. Market-based indicators of systemic risk enable monitoring how such weakness emerges and progresses in real time. In this paper, we focus on 1 such indicator, SRISK, which stands for systemic risk and measures the dollar amount of capital that a financial firm would need to raise to function normally if we have another financial crisis based on stock market data.

Using the Romer–Romer crisis severity measures (1), we propose a model to estimate the level of undercapitalization that precipitates a financial crisis. We calculate the probability of a crisis as a function of the aggregate capital shortfall and other variables for 23 developed economies over time. From these estimates we can then compute the SRISK capacity which would keep this probability below 50% as long as SRISK remains below this level.[†]

Our analysis features 2 widely recognized externalities. The risk of a financial crisis in a country depends on the total capital shortfall of the financial sector in this country. Thus, a firm that increases its leverage or beta will not only increase its own risk but also increase the risk of other financial institutions in the country. This incentivizes risk taking. Similarly, the risk of any one country depends on the aggregate SRISK of the rest of the world. Hence, a country that relaxes its regulation or fails to adequately capitalize its institutions will increase the risk of a financial crisis in other countries. This global externality clearly calls for a coordinated approach for regulation to maintain financial stability.

Most closely related to our paper, Adrian et al. (3) study how financial conditions affect the entire conditional distribution of gross domestic product (GDP) growth using a quantile regression approach. They find that financial conditions forecast downside risk to GDP growth. Ref. 4 extends this growth-at-risk analysis to the international context and to the term structure setting. A similar quantile regression approach is also used by ref. 5 for evaluating the ability of various empirical measures of systemic risk to predict real activity outcomes. Our paper differs in several ways. First, we focus on financial crises featuring a disruption in credit supply and do not attempt to model economic output whose lower tail can be associated with a wide range of distinct phenomena. Moreover, we calculate the probability of a financial crisis from real-time indication of excessive credit growth. Finally, we model and test the cross-border externality of financial undercapitalization in our empirical investigation and document its significant role.

Excessive Credit Growth

It is widely believed that financial crises result from excessive credit growth. See, for instance, ref. 6, which claims that this time

Significance

This study develops quantitative estimates of the level of systemic risk in the financial sector that precipitates a financial crisis. When financial firms are undercapitalized, they face difficulty in covering losses in a downturn. The natural response to such vulnerability, reducing leverage through asset sales, can start a financial crisis. Perilous excessive credit growth is reflected in the undercapitalization of the financial sector. Market-based indicators of systemic risk such as SRISK, which stands for systemic risk, measure such weakness in real time. We develop a probability of crisis measure and an SRISK capacity measure for 23 developed countries. Our analysis highlights the important global externality whereby the risk of a crisis in one country depends on the undercapitalization of the rest of the world.

Author contributions: R.F.E. and T.R. designed research, performed research, analyzed data, and wrote the paper.

Reviewers: T.A., International Monetary Fund; and J.D.D., Stanford University.

The authors declare no conflict of interest.

Data deposition: The statistical analysis code used in this paper has been deposited in the Harvard Dataverse (https://doi.org/10.7910/DVN/LH3RRR).

[1]To whom correspondence may be addressed. Email: rengle@stern.nyu.edu or tianyue.ruan@nus.edu.sg.

This article contains supporting information online at www.pnas.org/lookup/suppl/doi:10.1073/pnas.1903879116/-/DCSupplemental.

Published online August 27, 2019.

[†]This paper previously circulated with the title "How Much SRISK Is Too Much?" Some of the results are surveyed in the review article by Engle (2).

is not different; refs. 7 and 8 on financial cycles; refs. 9 and 10 on leverage cycles; and ref. 11 on the predictive power of credit growth.

A practical challenge of applying these findings for real-time monitoring of financial crisis risk is how to measure excessive credit growth. We argue that credit growth is excessive if the financial sector does not have sufficient capital to cover market value losses in a downturn. This is consistent with the notion that at the end of a credit cycle, increasingly risky credit will be issued and the holders of this credit will be leveraged financial institutions with insufficient capital to cover losses in a downturn.[‡] This is how a "credit boom goes bust" (11).

Our approach to distinguish between productive credit growth and excessive credit growth focuses on the quality and risk of firms extending and ultimately holding credit.

Consider a simple example of a bank that holds mortgages. This bank may eventually lend to underqualified borrowers and overvalued houses at the end of a credit cycle. In this case, the mortgages will actually be worth less than the accounting values and the bank's ratio of market value to book value will fall. This can be seen in the equity value of the firm as well as the mark-to-market valuation of the loans. If there is a downturn in the housing market, the value of the mortgages will fall farther as the collateral loses value and the borrowers become weaker. The firm may have to cover these losses from its capital, and if the capital is inadequate, the value of the assets of the bank may fall below its liabilities. In this case, stock market valuation will collapse, and the bank may seek a bailout to continue functioning. Thus excessive credit growth can be measured by the capital shortfall of the financial sector.

From this example, it is natural to examine the valuation of financial sector assets relative to their liabilities. If the assets are undervalued and risky, we would expect to see low market-to-book ratios and high equity volatility. If this is a systemic problem as opposed to an idiosyncratic one, we would expect to see high correlations with market-wide events. This is measured by the stock market beta of the firm. In a stress scenario, the broad market index will decline, and the impact on each firm's capital will depend on its beta. And in fact, the betas will differ depending on the asset holdings as the stock market is well aware of these effects.

The deterioration in credit quality during excessive credit growth and the adjustment in market valuation are succinctly reflected in a measure of capital shortfall under stress.

Developed by refs. 12–15, SRISK measures the dollar amount of capital that a financial firm would need to raise to continue to function normally if we have another financial crisis from stock market data. Because it is difficult to raise capital in a financial crisis, either this capital shortfall will be met mostly by the taxpayer money or the firm will cease to function normally and may fail. For this reason, the measure is considered to be an indicator of systemic risk in much the same way as are supervisory stress tests.

Normal operation of a financial firm requires that its market capital ratio (its market value of equity divided by the sum of the book value of liabilities and the market value of equity) be above the prudential capital ratio. Let k denote this prudential ratio. The capital shortfall of a financial firm at time t can therefore be computed as

$$Capital\ Shortfall_t = k\,(Debt_t + Equity_t) - Equity_t.$$

SRISK is defined as the median capital shortfall conditional on a future financial crisis:

$$\begin{aligned} SRISK_t &= \mathrm{M}_t\,(Capital\ Shortfall_{t+T}|Crisis_{t+T}) \\ &= \mathrm{M}_t\,(kDebt_{t+T} - (1-k)Equity_{t+T}|Crisis_{t+T}). \end{aligned}$$

Under the simplifying assumption that the debt value remains constant, SRISK boils down to the equity valuation conditional on a financial crisis. Under some mild assumptions, the formula for SRISK for a financial institution is given below:

$$SRISK_t = kDebt_t - (1-k)Equity_t \exp\left(\tilde{\beta}_t \log(1-\theta)\right). \quad [1]$$

We explain the key parameters and ingredients here and leave the full details to *SI Appendix*, section S1. $\tilde{\beta}_t$ is the beta coefficient from the dynamic conditional beta (DCB) model (16) which augments a standard market model with asynchronous trading, time-varying correlation, and asymmetric volatility. k is set at 8% which corresponds to the typical leverage ratio of well-managed financial firms in tranquil periods.[§] We consider the crisis to be 6 mo in the future and calibrate the market stress level θ to be 40% as the MSCI ACWI index declined ∼40% over 6 mo during the global financial crisis. For insurance companies, we make an adjustment for "separate accounts" which correspond to insurance clients' investments. In line with our calibration of prudential capital ratio from calm times, we include 40% of separate accounts to calculate SRISK for insurance companies. For each country, the aggregate SRISK is the sum of all of the financial firms with positive values.

Deleveraging Cycles

When undercapitalization as reflected in SRISK is high, either the regulator or the risk manager of individual companies will compel the firms to strengthen their balance sheets. To return to the target capital ratio, an undercapitalized financial firm can obtain more capital either by issuing new shares or by selling assets and using the proceeds to reduce debt. When sales of new shares or assets occur in large volume, it is inevitable that the transaction price declines and such an unfavorable price movement hinders the effectiveness of the deleveraging efforts.[¶] In the context of asset sales, such a price impact is also commonly referred to as a "fire sale externality." This can lead to a downward spiral of the financial sector and ultimately of the economy that has been written about extensively (for example, refs. 17–19). The initial conditions that make such a fire sale likely are precisely a large quantity of goods to be sold in a hurry without sufficiently many willing buyers. In this paper, we focus on operationalizing these concepts and derive a measure of the severity of systemic risk for each of the deleveraging strategies.

If the undercapitalized firms choose to issue new shares to raise capital, they may lower the values of existing shares. That is partly a signaling effect whereby the signal that a bank must raise capital conveys information that it is in trouble. It may also simply be a supply and demand effect where the more shares are in existence, the lower the price. Reducing payouts in the form of dividends or share repurchases has similar effects. In either case, if the value of shares that need to be sold is a large fraction of the shares that are already outstanding, the price impact is likely to be substantial, and firms will hesitate to use this channel. Hence a natural measure of excessive risk is SRISK/MV, where MV is the market capitalization.

[‡]This approach to financial crisis risk monitoring is previously reported in ref. 2.

[§]Firms using IFRS accounting rather than GAAP typically have a bigger balance sheet as there is less netting of derivatives. Consequently, we use 5.5% for all European firms.

[¶]Some of the details of these deleveraging cycles, elaborated below, are previously reported in ref. 2.

An alternative for deleveraging is to sell assets and use the proceeds to pay back debt. If the firm chooses this route and the total amount contemplated is small compared with the stock of assets, then asset sales are likely to be cost effective. Large asset sales, however, have a damaging impact in that they are likely to depress the price of assets which will, in turn, increase the leverage of all financial firms holding similar assets. Such a phenomenon is often called a leverage spiral in that deleveraging by selling assets at a minimum will require more sales than initially anticipated and may even be counterproductive in the extreme. Frequently this leverage spiral is called a "fire sale" and may lead to asset sales at prices below their fundamental value. In the market microstructure literature, a similar phenomenon is described as price impact: Market participants reduce their expectations of future value when they observe selling activities and therefore lower the price. In either context, there are many sellers and insufficient quantities of motivated and well-capitalized buyers to prevent the price from declining.

The natural risk measure if firms choose to deleverage by selling assets is, therefore, the ratio of assets for sale over total assets. If there is no price impact, there would be no effect on equity, and therefore the amount of assets to be sold to repay debt to reduce SRISK to 0 equals SRISK/k.

When it amounts to a large fraction of total assets, the realized price impact is likely to be substantial and triggers additional asset sales and a downward spiral in the asset price in a more realistic setting. Thus a natural measure of the size of SRISK which is dangerous is SRISK/(TA$^*k$), where TA stands for the total assets in the financial sector. Consistent with our partial inclusion of separate accounts for calculating SRISK, only 40% of separate accounts are included in total assets.

Both equity issuance and asset sales can be ineffective in achieving the target capital ratio when they need to occur in a large volume. Thus, undercapitalized financial institutions may choose between these approaches depending on the ratio of assets to market cap in the financial sector as a whole. This measure of leverage implies that firms would be more likely to raise equity when leverage is low and sell assets when leverage is high. Since financial crises often coincide with periods of stock market crash and high leverage, the sale of assets is a common approach. Furthermore, the well-known debt overhang problem first described in the seminal paper by Myers (20) becomes more pronounced in these periods: The potential increase in the value of assets should a firm decide to hold on to its assets mostly accrues to existing debt holders. A firm that maximizes equity value may therefore choose to sell assets rather than raise capital to deleverage.

There is another possibility if the undercapitalized firms do nothing instead of deleveraging through equity issuances or asset sales. If the growth opportunity materializes, the firms regain capital adequacy; if it does not, the firms appeal for a government bailout. In this scenario, the cost to the regulator is the loss of GDP that would be required in a bailout. Thus, the natural measure of the size of the risk is SRISK/GDP. When this ratio is high, there is elevated risk to the economy.

Data and Econometric Specifications

Country-level SRISK data, as well as the total market capitalization and total banking assets, are obtained from New York University (NYU) Stern's Volatility Laboratory (V-Laboratory). SRISK has been available since 2000. GDP data are from the World Bank (21).

We adopt the Romer–Romer text-based measure of financial crisis severity (1). This is a semiannual measure of crisis severity derived from the OECD Economic Outlook that is available for 24 developed economies. According to its classification criterion, the primary feature underlying a financial crisis is a disruption in credit supply. This measure is on a scale of 0 to 15. If there is no crisis, the measure is 0. From 1 to 15, a financial stress goes from a "credit disruption minus" to "extreme crisis plus." If the measure is greater than 3, it becomes more than a "minor credit disruption." Therefore, this measure not only records whether there is a financial crisis but also assesses how severe the crisis is.

We use this crisis measure for 23 of the countries studied by Romer and Romer (1). We do not include Iceland because it still does not have any publicly traded banks. During the 2007 to 2009 global financial crisis, all of them failed. To align with the semiannual frequency of the Romer–Romer crisis severity measure, we average the monthly SRISK variables over each 6-mo period for each country.

The Romer–Romer measure is available for the period from 1967 to 2012. Therefore, we estimate the empirical model for the period from 2000 to 2012. During this period, there is substantial variation in whether a financial crisis occurs as well as the severity and duration of a financial crisis both across countries and over time.

We run a battery of specification tests to select which one(s) of the 3 scaled versions of SRISK that each corresponds to a different deleveraging strategy—SRISK/GDP for government bailout, SRISK/MV for equity issuance, and SRISK/(TA$^*k$) for asset sales—would be most useful to explain crisis severity. The results are reported in *SI Appendix*, section S2. From these tests, we find that SRISK/(TA$^*k$) is the most important variable.

A financial crisis represents a left tail event for the economy. Any measure of financial crisis severity does not distinguish between strong and borderline economic conditions as long as a crisis has not started yet. Therefore, such a measure represents a truncated indication of economic condition. The relationship between crisis severity and SRISK is naturally a hockey stick rather than a straight line. The Tobit model which recognizes that the dependent variable is truncated at 0 is the preferred estimator. We report linear regression results in *SI Appendix*, section S2.

In the Tobit model, a latent variable y_l is a linear function of explanatory variables X and a disturbance. The observed dependent variable, y, is a truncated version of y_l. Under the assumption that the error term follows a standard normal distribution, the model can be expressed by 2 equations as follows:

$$y = \begin{cases} y_l \text{ if } y_l > 0 \\ 0 \text{ otherwise} \end{cases}$$
$$y_l = X\beta + \sigma\varepsilon, \varepsilon \sim \mathcal{N}(0,1).$$

We estimate the Tobit model with country fixed effects to allow the possibility that countries will differ in the tolerable level of SRISK. This may be due to institutional markets for selling assets and pools of investors who might be willing to step in even as a crisis is approaching. It may also be due to differences in the likelihood of a government rescue that would protect both financial firms and those buying assets.

We consider a domestic model that uses only country-level SRISK variables to explain crisis severity and a global model that expands the set of explanatory variables with world SRISK variables. The motivation of the global model comes from the observed comovement in crisis severity across countries in *SI Appendix*, Fig. S1 and the correlation between world SRISK variables and the estimated time fixed effects in *SI Appendix*, Table S1. To better capture the externality aspect of financial crises, we modify how these world variables are constructed. For each country, the world SRISK variables are calculated using the sum of the respective country-level variables across all other countries, which we refer to as leave-out sums. This modification also facilitates the SRISK capacity measure developed later.

ECONOMIC SCIENCES

The estimation results are reported in Table 1. The SRISK/(TA$^*k$) variable is highly significant in either the domestic model or the global model. Columns 1 and 2 in Table 1 are the specifications with the best Schwarz criterion among many specifications including many not reported here for the domestic and global models, respectively.

Based on the Tobit model, we can assess the distance from a financial crisis quantitatively. We propose 2 measures for this quantitative assessment. The first one is a probability of a crisis. Following the Tobit model, such measure can be expressed as the probability that the dependent variable will exceed a value q conditional on X for a meaningful value of $q > 0$. We choose $q = 4$ which corresponds to a mild crisis under the Romer–Romer classification:

$$ProbCrisis = \Pr(y > 4 \mid X) = \Pr(y_l > 4 \mid X) = \Pr\left(\varepsilon > \frac{4 - X\hat{\beta}}{\hat{\sigma}}\middle| X\right) = 1 - \Phi\left(\frac{4 - X\hat{\beta}}{\hat{\sigma}}\right). \quad [2]$$

The second measure gauges whether there is a level of SRISK that makes the probability of a crisis just 50%. From Eq. **2** we see that the probability of a crisis is 50% when $X\hat{\beta} = 4$. We can solve for a SRISK capacity that corresponds to this level of risk, holding everything else constant. Here $\hat{\beta}_1$ is the estimated (combined) coefficient on SRISK/(TA$^*k$) in the Tobit estimation. In the domestic model, since both SRISK/(TA$^*k$) and its lag are included in the domestic model, $\hat{\beta}_1$ is the sum of their coefficients or 24.917. In the global model, $\hat{\beta}_1$ is the coefficient of country SRISK/(TA$^*k$) or 13.165:

$$SRISKCapacity = SRISK + \frac{4 - X\hat{\beta}}{\hat{\beta}_1} \times k \times TA. \quad [3]$$

We compute and analyze these 2 measures at the monthly frequency. To reconcile with the semiannual frequency of the estimation sample, we use the 6-mo moving average of the explanatory variables to construct these 2 measures.

Table 1. Crisis severity and systemic risk measures (Tobit)

	Romer–Romer crisis severity	
	1)	2)
SRISK/(TA*k)	18.325***	13.165***
	(1.213)	(1.366)
D.SRISK/(TA*k)	6.592***	
	(1.931)	
World SRISK/(TA*k)		14.249***
		(2.387)
D.World SRISK/(TA*k)		7.987***
		(2.759)
Var(e.CRISIS)	11.102***	9.852***
	(1.263)	(1.110)
Country fixed effects	Yes	Yes
Pseudo R^2	0.261	0.291
Observations	561	561

Shown are the Tobit estimates of how systemic risk measures are contemporaneously associated with crisis severity. The sample includes country half-year observations for all countries studied by ref. 1 with the exception of Iceland from the second half of 2000 to the second half of 2012. The world SRISK variables are calculated using leave-one-out sums. SEs are reported in parentheses. *** represents 1% significance. Reproduced with permission from ref. 2, *Annual Review of Financial Economics*, Volume 10, copyright 2019 by Annual Reviews, http://www.annualreviews.org.

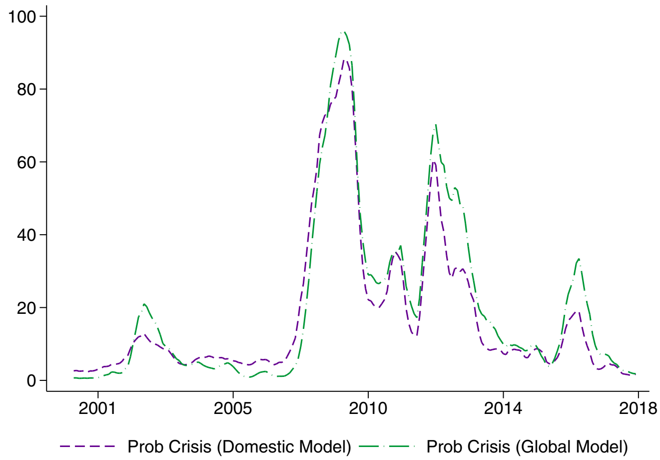

Fig. 1. Probability of crisis (%): United States. Reproduced with permission from ref. 2, *Annual Review of Financial Economics*, Volume 10, copyright 2019 by Annual Reviews, http://www.annualreviews.org.

Results and Discussion

We compute the probability of crisis and the SRISK capacity from both the domestic model and the global model.[#] Figs. 1 and 2 plot these 2 measures for the United States. Similar figures are included in *SI Appendix*, section S4 for all other countries.

From Fig. 1, we can see that the domestic and global models give rather similar estimates for the probability that the United States is in a crisis over the 17-y period although the peaks are a little higher in the global model. In 2008 the probability rose to 80% or 90% whereas it was only about 60% in the European sovereign debt crisis and in the recent period has fallen to less than 10%. In contrast, many of the European countries had a greater peak in 2011 than in 2008.

Fig. 2 shows the distance to a crisis for the United States. The red solid line is the SRISK for the United States; whenever this exceeds the SRISK capacity, the probability of crisis rises above 50%. The SRISK capacity obtained from the domestic model was rising from 2000 to 2009 and stayed relatively unchanged since then. On the contrary, the SRISK capacity obtained from the global model dived in 2008 to 2009, 2011 to 2012, and 2016. This difference means that the global model predicts a faster increase in the likelihood of a financial crisis for any given level of SRISK than the domestic model during these 3 periods. In a similar vein, the probability of crisis obtained from both models peaks at the same time. These 3 incidences, not unique to the United States, correspond to the 3 crisis episodes since 2000 and reflect the global nature of financial crises.

The time-series dynamics of world SRISK since 2000 (*SI Appendix*, Fig. S2) reveal 3 peaks. The magnitude of the peak is close to $4 trillion in each case and greatly exceeds the SRISK during the first 7 y of this century. The first 2 peaks correspond to 2 well-known crisis episodes, the global financial crisis and the European sovereign debt crisis. The third peak in 2016 to 2017 can be viewed as an Asian debt crisis with Japan and China as the main contributors. What is common in all 3 episodes is that banks massively increase their holdings of assets that are perceived to be riskless and subsequently experience stress in 1 or at most a handful of countries and that such financial stress spreads from these countries to other countries. We provide a more detailed description of these 3 crisis episodes in *SI Appendix*, section S3.

The global model captures the important global externality whereby the risk of a crisis in one country is strongly influenced

[#]Some of these results are previously reported in ref. 2.

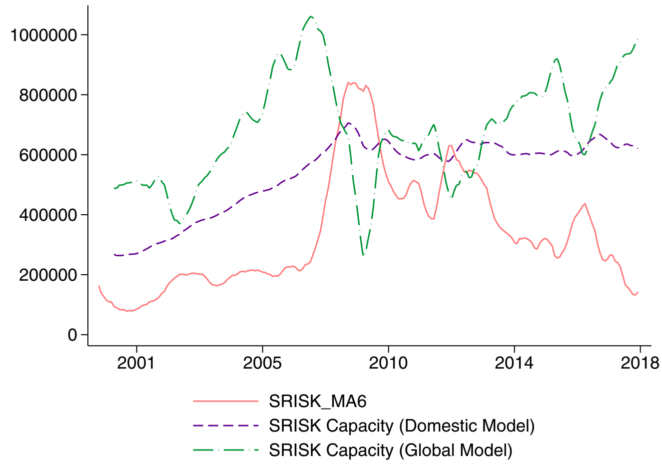

Fig. 2. SRISK capacity (USD million): United States. Reproduced with permission from ref. 2, *Annual Review of Financial Economics*, Volume 10, copyright 2019 by Annual Reviews, http://www.annualreviews.org.

by the rest of the world. The undercapitalization of the financial sector in one country will increase the probability of a crisis in another. Financial stability is interconnected across borders, and each country has a stake in the regulation of the rest of the world.

We also offer a discussion of our measurement choices here.

Market-based measures of systemic risk are useful for this analysis because of their forward-looking nature. The conditional value-at-risk (CoVaR) measure (22) is an alternative market-based measure that is closely related to SRISK. Under the simplifying assumption that returns of a firm and the broad market index are conditional bivariate normal, both CoVaR and the median equity valuation conditional on a financial crisis are linear in the correlation between the 2 returns (*SI Appendix*, Eqs. **S10** and **S11**). The main difference is that SRISK depends also on the firm's volatility whereas CoVaR does not due to differences in conditioning. In addition, SRISK depends on both size and leverage.

Besides the Romer–Romer chronology, several other crisis chronologies exist. Almost all of them use a 0 to 1 classification: Either a country experienced a crisis or it did not. Such binary classification does not distinguish between mild crises and severe crises. Moreover, these alternative chronologies are mostly done ex post and sometimes identify crisis episodes based on a wide range of distinct phenomena, such as asset price declines, banking problems, and consumer or business bankruptcies. The Romer–Romer methodology that compiles a continuous measure of disruptions of credit supply from real-time narrative accounts is suitable for our purpose.

Robustness

Parameter Stability. As a robustness check, we reestimate the domestic and global models excluding 1 country from the sample at a time. *SI Appendix*, Figs. S26 and S27 show the point estimates and the 95% confidence intervals associated with each main regressor for the domestic model and the global model, respectively. The horizontal axis lists the country that is dropped from the sample. For instance, the first point corresponds to the estimation without Australia, and the last point corresponds to the estimation without the United States. For each regressor, we also draw a horizontal line at the value of the coefficient obtained from the full sample of 23 countries. In *SI Appendix*, Figs. S26 and S27, we observe that the significance of all main regressors, as well as the magnitude, remains relatively unchanged no matter which country is excluded from the sample.

Varying Tuning Parameters. SRISK is calculated under a given stress level and a given prudential capital ratio. For insurance companies, there is another assumption on the included fraction of separate accounts. In our analysis so far, we have fixed these factors and estimated the coefficients of SRISK measures calculated under the fixed values. In this section, we relax the assumption on these factors and examine the impact of varying them. It turns out that the stress level, the prudential capital ratio, and the included fraction of separate accounts can also be thought of as parameters whose different values give rise to different models ("tuning parameters").

The firm-level SRISK depends on the prudential capital ratio and the included fraction of separate accounts linearly and on the stress level nonlinearly. Another source of nonlinearity comes from insurance companies; adjusting for separate accounts affects both the numerator and the denominator of SRISK/(TA$^*k$) variables. When we aggregate firm-level SRISK into country-level SRISK, we sum across firms with positive SRISK, which makes country-level SRISK nonlinear on all these 3 tuning parameters. In light of the nonlinearity, we consider a grid of tuning parameter values and iterate through the grid to find the best model:

- Stress level (θ) ranging from 30% to 60% in a 5% increment.
- Included fraction of separate accounts (s) ranging from 0% to 100% in a 20% increment.
- Capital ratios (k) can take 5 different pairs of value. Currently, we assume k2 = 5.5% for European countries and k1 = 8% for all other countries. The other 4 values assume the same treatment for all countries: 4%, 5.5%, 8%, and 10%.

Overall, there are $7 \cdot 6 \cdot 5 = 210$ combinations. For each combination, we recalculate firm-level SRISK using Eq. **1** and get country-level SRISK by summing all positive-SRISK firms for each point in time and reestimate both the domestic model and the global model.

A natural criterion we can use to select the best model is the highest (maximized) log-likelihood.[‖] Based on this criterion, the best domestic model is achieved with a stress level of 50%, an included fraction of separate accounts of 0%, and a capital ratio of 5.5% for all countries; the best global model is achieved with a stress level of 60%, an included fraction of separate accounts of 0%, and a capital ratio of 4% for all countries.

In both cases, the selected best model is different from the baseline model. This "in sample" victory, however, can be driven by the realized sample of data or be truly indicative of a difference "in population." To tell these 2 possibilities apart, we also need to compare the relative performance of the models more rigorously by assessing whether the difference in performance is statistically significant based on the work of Diebold and Mariano (23).

We conduct panel Diebold–Mariano tests using 3 different heteroscedasticity and autocorrelation consistent (HAC) SEs. For the domestic models, we find that regardless of which HAC SEs are used, the difference from the baseline model is insignificant for any alternative model. Therefore, we conclude that the baseline model is adequate. For the global model, there is evidence that better stress tests can be found than the baseline when using the simple Newey–West SEs but these are not significantly different when using measures which take the panel structure into account. We provide an in-depth description of the methodology and the test results in *SI Appendix*, section S6.

‖Since the number of parameters estimated in the Tobit models and the sample period stay the same when we vary the values of these tuning parameters, selecting the best model with AIC or BIC yields the same result.

ECONOMIC SCIENCES

Conclusion

We have estimated a model of systemic risk which is designed to show both the probability of a crisis and the distance between current measures of systemic risk and the level which makes the probability of crisis equal to one-half. The model builds on the theory that deleveraging will have a price impact and the greater the magnitude of the deleveraging the more dangerous the adjustment. In its most extreme case, the real economy has restricted access to credit as the financial sector experiences a fire sale, thus endogenously generating a financial crisis.

This paper quantifies this process with a simple model that incorporates systemic externalities both within countries and between countries. These externalities reinforce the potential benefits of financial regulation and coordination on a country and an international level.

Countries can and do differ in their tolerance of financial undercapitalization, although we do not explore the economic or political origins of these differences. The main results are insensitive to dropping any of the countries in the sample and to alternative parameters of the stress tests. Thus we hope that this research will provide a reliable guide to how much systemic risk is too much.

ACKNOWLEDGMENTS. We thank the Sloan Foundation, the National Science Foundation (NSF), the Macro Financial Modeling (MFM) Group under the leadership of Lars Hansen and Andy Lo, and NYU Stern for financial support to the Volatility Institute. We further thank the Armellino Foundation, BlackRock, the Global Risk Institute, the Norwegian Finance Initiative, the Inter-American Development Bank, and Investment Technology Group for additional support. We are indebted to many seminar participants from the Bernoulli Lecture at École Polytechnique Fédérale de Lausanne, Sapienza University in Rome, the Society of Financial Econometrics (SoFiE) 10th Annual Conference, the Slovak Economic Association, Central European University, Hong Kong Polytechnic University, Zhejiang University, the Volatility Institute at NYU Shanghai, International Finance and Banking Society (IFABS) Asia 2017 Ningbo China Conference, the Inter-American Development Bank, the National Bureau of Economic Research/NSF Time Series Conference at Northwestern University, the Royal Danish Academy, the MFM Winter 2018 Meeting, and NYU Stern. Particularly helpful comments from Darrell Duffie, Eric Jondeau, Diane Pierret, Marina Brogi, Lasse Pedersen, Soren Johansen, Katarina Juselius, Mark Gertler, Ed Altman, Richard Berner, Kim Schoenholtz, Viral Acharya, Ralph Koijen, Markus Brunnermeier, Tobias Adrian, Matt Richardson, Thomas Philippon, Philipp Schnabl, and many others are deeply appreciated.

1. C. D. Romer, D. H. Romer, New evidence on the impact of financial crises in advanced countries. *Am. Econ. Rev.* **107**, 3072–3118 (2017).
2. R. Engle, Systemic risk 10 years later. *Annu. Rev. Financ. Econ.* **10**, 125–152 (2018).
3. T. Adrian, N. Boyarchenko, D. Giannone, G. Domenico, Vulnerable growth. *Am. Econ. Rev.* **109**, 1263–1289 (2019).
4. T. Adrian, F. Grinberg, N. Liang, S. Malik, The term structure of growth-at-risk. *International Monetary Fund* (2018). https://www.imf.org/en/Publications/WP/Issues/2018/08/02/The-Term-Structure-of-Growth-at-Risk-46150. Accessed 20 August 2019.
5. S. Giglio, B. T. Kelly, S. Pruitt, Systemic risk and the macroeconomy: An empirical evaluation. *J. Financ. Econ.* **119**, 457–471 (2016).
6. C. Reinhart, K. Rogoff, *This Time is Different: Eight Centuries of Financial Folly* (Princeton University Press, 2009).
7. C. Borio, The financial cycle and macroeconomics: What have we learnt? *J. Bank. Financ.* **45**, 182–198 (2014).
8. M. Drehmann, C. Borio, K. Tsatsaronis, Characterising the financial cycle: Don't lose sight of the medium term! *BIS* (2012). https://www.bis.org/publ/work380.htm. Accessed 20 August 2019.
9. T. Adrian, H. S. Shin, Liquidity and leverage. *J. Financ. Intermediation* **19**, 418–437 (2010).
10. T. Adrian, H. S. Shin, Procyclical leverage and value-at-risk. *Rev. Financ. Stud.* **27**, 373–403 (2014).
11. M. Schularick, A. M. Taylor, Credit booms gone bust: Monetary policy, leverage cycles and financial crises, 1870–2008. *Am. Econ. Rev.* **102**, 1029–1061 (2012).
12. V. V. Acharya, C. T. Brownlees, R. F. Engle, F. Farazmand, M. Richardson, "Measuring systemic risk" in *Regulating Wall Street: The Dodd-Frank Act and the New Architecture of Global Finance*, V. V. Acharya, T. F. Cooley, M. P. Richardson, I. Walter, Eds. (Wiley, 2011), pp. 87–119.
13. V. V. Acharya, R. F. Engle, M. Richardson, Capital shortfall: A new approach to ranking and regulating systemic risks. *Am. Econ. Rev.* **102**, 59–64 (2012).
14. C. T. Brownlees, R. F. Engle, Volatility, correlation and tails for systemic risk measurement. *SSRN* (2011). https://papers.ssrn.com/sol3/papers.cfm?abstract_id=1611229. Accessed 20 August 2019.
15. C. T. Brownlees, R. F. Engle, SRISK: A conditional capital shortfall index for systemic risk measurement. *Rev. Financ. Stud.* **30**, 48–79 (2017).
16. R. F. Engle, Dynamic conditional beta. *J. Financ. Econom.* **14**, 643–667 (2016).
17. R. Cont, E. Schaanning, Fire sales, indirect contagion and systemic stress-testing. *SSRN* (2016). https://papers.ssrn.com/sol3/papers.cfm?abstract_id=2541114. Accessed 20 August 2019.
18. R. Greenwood, A. Landier, D. Thesmar, Vulnerable banks. *J. Financ. Econ.* **115**, 471–485 (2015).
19. L. H. Pedersen, When everyone runs for the exit. *Internat. J. Cent. Banking* **5**, 177–196 (2009).
20. S. C. Myers, Determinants of corporate borrowing. *J. Financ. Econ.* **5**, 147–175 (1977).
21. New York University (NYU) Stern's Volatility Laboratory (V-Laboratory) Systemic Risk Analysis, Systemic Risk Analysis (Global Dynamic MES) of World Financials. https://vlab.stern.nyu.edu/analysis/RISK.WORLDFIN-MR.GMES. Accessed 3 May 2018.
22. T. Adrian, M. K. Brunnermeier, CoVaR. *Am. Econ. Rev.* **106**, 1705–1741 (2016).
23. F. X. Diebold, R. S. Mariano, Comparing predictive accuracy. *J. Bus. Econ. Stat.* **13**, 253–265 (1995).